

# Modal Analysis – Experimental Set-Up to Determine the Floor Dynamics at IIT Kharagpur

**Shashi Shekhar Singh**  
Assistant Professor  
Department of Civil Engineering  
MVN University, Palwal, Haryana

**Ayush Srivastava**  
UG Student  
Department of Civil Engineering  
MVN University, Palwal, Haryana

**Avinash Kumar**  
Assistant Professor  
Department of Mechanical Engineering  
Satya College of Engineering and Technology, Palwal,  
Haryana

**Pankaj Kaushik**  
PG Student  
Department of Civil Engineering  
MVN University, Palwal, Haryana

## Abstract

Modern structures are becoming more and more slender but with improved stiffness with use of large number of materials in combination, making the predictions of dynamic characteristics of such structures extremely difficult for the analysis using commercially available general purpose finite element softwares. On the other hand, if a building floor is properly excited and the input force and the resulting responses are accurately measured with proper hardware, the dynamic characteristics can be found out without much error. This work is focused on complete experimental setup of actual physical measurement of input force of a reaction mass shaker to a typical building floor to determine the floor dynamics.

**Keywords:** Structural Analysis, EMA – Experimental Modal Analysis, FRF – Frequency Response Function, FEM - Finite Element Method, Data acquisition, MEScope

## I. INTRODUCTION

Structural Analysis, EMA – Experimental Modal Analysis, FRF – Frequency Response Function, FEM - Finite Element Method, Data acquisition, MEScope.

### A. Derivation of the Response Model:

The matrix equation of motion for the forced vibration of a viscously damped system is given by:

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = \{f(t)\} \quad (1)$$

It can be shown that it is possible to calculate the response of the structure to a system of forces using the equation:

$$\{X(t)\} = ([K] - \omega^2[M] + i\omega[C])^{-1} \{f(t)\} \quad (2)$$

which may also be written as:

$$\{X\} = [\alpha(\omega)]\{f\} \quad (3)$$

where:

$$[\alpha(\omega)] = ([K] - \omega^2[M] + i\omega[C])^{-1}$$

### B. Use of the Modal Model:

It is more usual in EMA to define the response model in terms of its modal properties, rather than its spatial properties, in order to simplify the mathematics. Unfortunately, the modal solution of “1” is difficult since a non-proportional viscous damping matrix [C] serves to make the eigenvalues and eigenvectors complex. For this reason, it is more usual in EMA to utilize the ‘hysteretic damping’ formulation. In this case, the matrix equation of motion is given by

$$[M]\{\ddot{x}(t)\} + ([K + iD])\{x(t)\} = \{f(t)\} \quad (4)$$

Where the ‘proportional’ hysteretic damping matrix [D] is given by:

$$[D] = a_1 [M] + a_2 [K] \quad (5)$$

The receptance matrix may therefore be expressed as

$$[\alpha(\omega)] = [\phi] \left[ \begin{matrix} \backslash \\ (\omega_r^2 - \omega^2) \\ \backslash \end{matrix} \right]^{-1} [\phi]^T \quad (6)$$

Any individual element in the receptance matrix is a single FRF and may be calculated from:

$$\alpha_{jk}(\omega) = \sum_{r=1}^N \frac{(\phi_{jr})(\phi_{kr})}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2}$$

or

$$\alpha_{jk}(\omega) = \sum_{r=1}^N \frac{{}_r A_{jk}}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2} \quad (7)$$

Where  ${}_r A_{jk} = (\phi_{jr})(\phi_{kr})$  is called the modal constant and  $\eta_r$  is known as the ‘loss factor’ for mode r. It can be shown that, at frequencies close to resonance, the loss factor has a value approximately double that of the equivalent modal viscous damping ratio, i.e.  $\eta_r \approx 2\zeta_r$ . It is, in fact, FRF relationships based on “7” which are most commonly used to obtain the modal parameters of a structure from experimental testing. A number of FRFs are calculated using direct force and structural response measurements, after which curve fitting is performed to estimate the structural modal properties.

$$[\alpha(\omega)] = ([K] - \omega^2 [M] + i\omega [C])^{-1}$$

is defined as the  $N \times N$  receptance matrix for the system which constitutes its response model. It should be noted that similar relationships may be derived relating the force input to the velocity and acceleration responses. In these cases, the receptance matrix is replaced by a mobility or inertance matrix respectively. The generic term ‘FRF matrix’ is frequently used to represent the response model given by “3” and it is normally represented as  $H(\omega)$ .

Since the force input to the structure and the dynamic response of the structure are amenable to physical measurement, it is theoretically possible to obtain a mathematical description of the structure through testing. The challenges involved in obtaining the force and response data, and of converting these data into a meaningful form, are at the heart of EMA.

## II. DYNAMIC SIGNAL PROCESSING

Most EMA is based on the calculation of FRFs from force and response signals, followed by curve fitting techniques which aim to determine modal properties, such as those featuring in “7”. However, the exact method of calculation of the FRF from the force and response time domain signals depends on the types of signals in question.

### A. Periodic Signals:

A periodic signal is a deterministic signal which repeats itself in time every T seconds, where T is known as the repeat period. Such a signal may be expressed in terms of a Fourier series:

$$X(t) = \sum_{p=-\infty}^{\infty} X_p e^{ip\omega_o t} \quad (8)$$

Where  $X_p$  is the complex Fourier coefficient given by:

$$X_p = \frac{1}{T} \int_t^{t+T} X(\tau) e^{-ip\omega_o \tau} d\tau \quad (9)$$

And  $\omega_o$  is the lowest frequency component of the signal given by

$$\omega_o = 2\pi f_o = 2\pi \frac{1}{T}$$

It can be seen that all frequency components of this signal ( $p\omega_o$ ) are discrete at integer multiples of  $\omega_o$ , which describe its ‘linear spectrum’. For modal testing applications, a dormant linear structure excited by a periodic forcing function will exhibit a periodic response (after any start up transients have died out) with the same period as the forcing function. It is therefore clear

that by measuring the forcing function and structural response over a single period, it is possible to determine the linear spectra of both signals and subsequently calculate an FRF “3” using complex division.

### B. Calculation of Frequency Response Functions:

Calculation of FRFs can be done through the use of FRF ‘estimators’ which, provided the force and response signals are processed in the same way, are applicable to all types of signals. One of the most common FRF estimators is the H1 estimator. The frequency domain input output FRF relationship for a linear system is given by:

$$Y(\omega) = H(\omega).X(\omega) \quad (10)$$

Where  $X(\omega)$  is the system input and  $Y(\omega)$  is the system output. Multiplying by  $X^*(\omega)$  gives:

$$X^*(\omega)Y(\omega) = H(\omega).X(\omega)X^*(\omega) \quad (11)$$

Which, assuming that both channels are processed in the same way, can be rewritten as:

$$S_{XY}(\omega) = H(\omega)S_{XX}(\omega) \quad (12)$$

The H1 estimator is therefore defined as:

$$H_1(\omega) = \frac{S_{XY}(\omega)}{S_{XX}(\omega)} \text{ for } -\infty \leq \omega \leq +\infty \quad (13)$$

Or:

$$H_1(\omega) = \frac{G_{XY}(\omega)}{G_{XX}(\omega)} \text{ for } 0 \leq \omega \leq +\infty \quad (14)$$

The H1 estimator was implemented in the spectrum analyser utilised in this study. In addition to being applicable to all types of signals, it also has a strong advantage in that the effects of environmental or instrumentation noise, present only on the response channel and uncorrelated with the force channel, will tend to reduce with averaging.

## III. MODAL PARAMETER ESTIMATION

The FRF matrix for a structure may be expressed in terms of its modal properties. Modal parameter estimation is a set of techniques by which the modal properties of the structure may be determined from part or the entire FRF matrix. In the experimental work carried out, the software ‘MEScope’ was used for performing required modal parameter estimation.

## IV. BASIC MEASUREMENT SYSTEM

The measurement system used in this work can be broadly divided into three major items based on its availability in the laboratory.

- 1) Excitation Mechanism
- 2) A transduction system( for measurement of force and response)
- 3) An analyzer to process the signals and give desired output

### A. Excitation and Data acquisition system used for Experimental Modal Analysis:

The excitation for the experimental work was provided by an APS Dynamics model 113 ES electrodynamic shaker (Spectral dynamics). It delivers a max force of 62 N, max Velocity 760 mm/s, max stroke 158mm and weight 38 kgs. Henceforth it will be termed as long stroke shaker/ shaker. This shaker system along with the data acquisition system was purchased during the course of this work and therefore the undersigned was responsible for commissioning of the same. For the same reason, a little elaboration will be made on the use of this system.

The long stroke shaker utilizes a current carrying conductor located within a dc magnetic field to generate a force, which is proportional to the instantaneous value of the supplied electric current. It is therefore possible to generate a time varying force of any form, within the force envelope of the shaker on availability of a controller. In the present system used for experiment, the FFT analyzer (OR 36, 4-16 channels) from OROS was used to generate a wave form in an open loop system through an amplifier.

Two methods to measure the force:

#### 1) Indirect Method:

The shaker was operated in free armature mode with reaction masses fixed to the armature of the shaker.

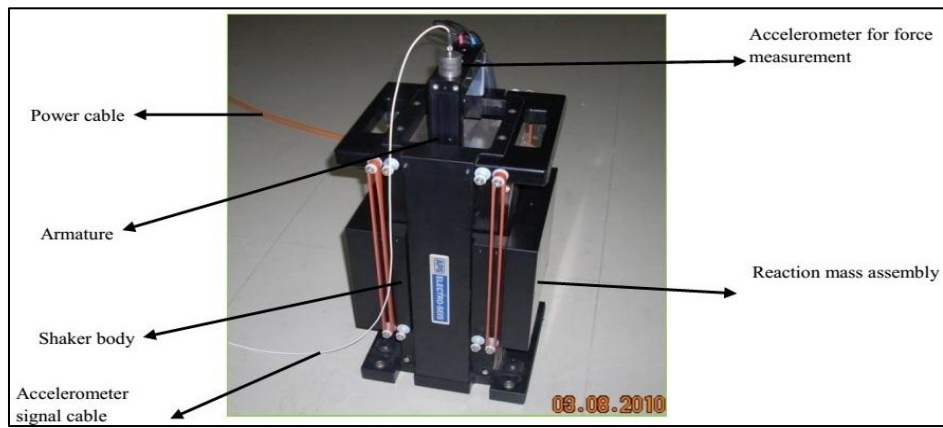


Fig. 1: Free Armature Mode of Shaker (Indirect Method)

The operation of shaker in free armature mode entailed placing of the shaker on the slab. The force was generated by accelerating reaction masses attached to the shaker armature, hence exerting an equal and opposite force to the shaker body and consequently to the structure itself. The force input to the structure was measured by the acceleration of the shaker armature and by multiplying it by the mass of the combined armature and reaction mass assembly. It was assumed that the mass of the shaker body was negligible compared to that of the test structure and also the acceleration of the shaker body was assumed negligible compared to that of the armature.

2) *Direct Method:*

The shaker was operated in free armature mode with reaction masses fixed to the armature of the shaker.

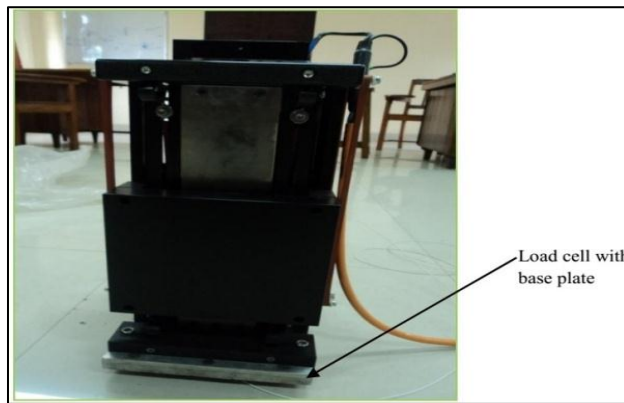


Fig. 2: Free Armature Mode of Shaker (Direct method)

The operation of shaker in free armature mode entailed placing of the shaker on the base plate. The force was generated by direct load cell.

**B. Response Transducers and Analyzer:**

Accelerometers (Model 393B04) with sensitivity of 1000 mV/g PCB piezoelectronics were used for acquiring the data. The accelerometers are piezoelectric sensors and have enough sensitivity to pick up floor vibrations. The data was transmitted using a coaxial cable connected to the accelerometer and the analyzer. The accelerometer was mounted on a circular base plate with leveling capability (Fig. 3). Since the acceleration of the floor was very less (less than 1g), no attempt was made to fix the accelerometers firmly to the ground.



Fig. 3: Piezoelectric Sensor with Base Plate

Load cell with sensitivity of 1000 mV/N PCB piezoelectronics were used for acquiring the force data. The load cell is piezoelectric sensors and has enough sensitivity to pick up force data. The data was transmitted using a coaxial cable connected to the load cell and the analyzer. The load cell was mounted on a rectangular base plate (Fig. 4).

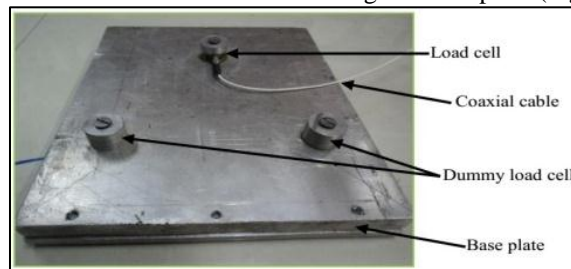


Fig. 4: Load Cell with Base Plate

The balance instruments along with the analyzer are as shown in Fig. 5. It consists of OROS analyzer which has got a capability of housing 16 input channels and 2 output channels to give excitation signal in desired waveform. The OROS analyzer was used to sample digitally the force and response signals and to perform immediate calculation of FRFs. The FRF data was stored in the analyzer itself. The OROS also has the capability to operate in analyzer mode and office mode where the real time data can be stored in the hard disc of the OROS and can be used for post analysis away from the field. However this is useful when measurement over a longer period of time is taken viz. ambient excitation. To carry out analysis, software named NV Gate was used which was compatible with OROS. The entire data acquisition setup is as shown in Fig. 5.



Fig. 5: Data Acquisition Setup

### C. Type of Excitation Signal:

The following signals were available with OROS analyzer to be used for exciting the shaker

- 1) Sine
- 2) Random
- 3) Chirp
- 4) Multisine

Sine waveform could not be used since a range of frequencies were required. Exciting the floor at each natural frequency as per the pre test analysis would be time consuming and may not fetch accurate results. Random signal with white and pink noise were available, but it was seen that the spectrum of the signal was not as desired. The energy dissipation throughout the range was not equal. Moreover it was difficult to window the signal using force weighting window. The chirp type of excitation provided better results. This is a fast swept sine waveform giving a near steady state response.

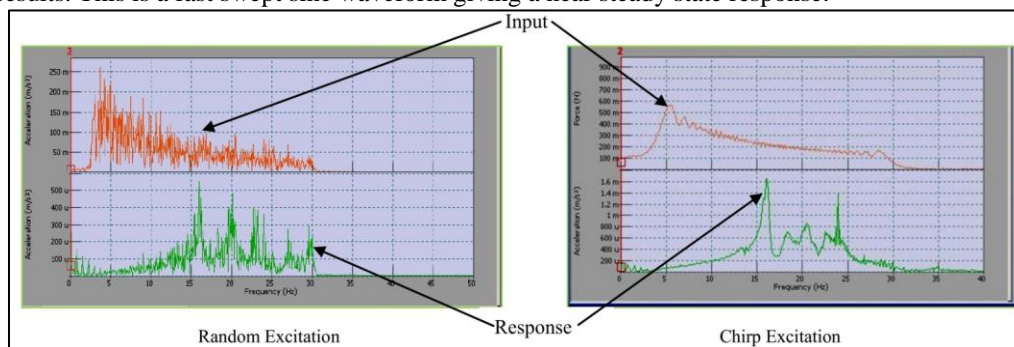


Fig. 6: Comparison of Random and Chirp excitation

In the course of experiment, it was also found that this type of excitation also gave very good force spectrum. By evaluating the FRF ordinates of all frequencies of interest, a complete FRF at a point was generated. No further try was made to improve the

random signal. It can be tried in future to see suitability of Random signal for excitation using a controlled vibration. Multi sine waveform was however not tried.

## V. EXPERIMENTAL MODAL ANALYSIS PROCEDURE

The Experimental Modal Analysis which was followed in this work was structured to follow certain procedures so that high quality data is recorded. This was necessary to save on time from taking the recording again. For the same, the procedures were broadly broken into four major parts

### A. Preparatory Phase:

This phase consisted of preliminary work like setting up of objective, doing some pre-test analysis, marking the structure for data acquisition, etc.

### B. Logistics Phase:

The proper functioning of all equipment was checked prior to departure to the test site. Packing was performed with the aid of checklists to ensure that nothing was forgotten. Wherever possible, spare parts were taken for critical items of equipment at the site. Extreme care was taken while shifting the shaker. The reaction masses and the outer bands were taken out, the internal bands were retained. The entire set of the shaker was installed at site. The OROS was used to give the excitation signal to the shaker. Care was also taken that the connections were checked before starting any instrument. The main parameters which were considered for logistics were:-

- 1) Approach to the slab or test floor without damaging the entire set.
- 2) Carriage of proper covers to site in case of rain
- 3) Availability of electricity at site.
- 4) Proper packing of equipment during transportation
- 5) Minor additional things like table, chairs, screw driver set etc.

Whilst the arranging of accommodation and transport for a site test may appear a trivial matter, it should nevertheless be given proper consideration.

### C. Exploratory Phase:

In this phase certain pretest measurements were carried out in order to check the suitability of the structure for data acquisition and also the correctness of the instrument at site.

### D. Measurement Phase:

In this phase the data acquisition was carried out and all FRF data obtained.

### E. Post Analysis Phase:

This phase was carried out to extract the modal parameters from the obtained FRF data. This was done entirely after the data acquisition was complete. An additional software MEScope was used for the same. The FRF data was imported from analyzer to the MEScope.

## VI. CONCLUSION

The Experimental modal analysis conducted on flooring systems show a very low frequency range of around 10 Hz – 50 Hz as compared to mechanical parts which are of few KHz. This experimentation is on as built floors with real time boundary conditions. The chirp type of excitation provided better results. The excitation of floors in practice is generally done using heel drop, instrumented sledge hammer or electrodynamic shaker. Since the flooring system prototypes are usually large, proper excitation is necessary to excite the floor to at least its first few natural frequencies, and more importantly to excite the floor in its global mode. We observed that the present system is suitable to excite long span floors to its global modes. Although more stress has to be laid on logistics with this system as compared to that of instrumented sledge hammer, the results are encouraging.

## REFERENCES

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