Five Dimensional Cosmological Model with Time Dependent Equation of State

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Abstract

In this work we have obtained a homogeneous cosmological model in higher dimensions by assuming time dependent equation of state p = f (t) with cosmological constant of the form $\Lambda = \alpha \frac{\dot{R}^2}{R^2}$ in the context of Kaluza Klein theory of gravitation. The solutions obtained in this work are realistic in the sense that physical properties, such as mass, density remain positive throughout the entire evaluation of the universe.

Keywords: Cosmology, equation of state, Einsteain Field Equation, Five dimensional Model, gravitation

I. INTRODUCTION

The application of extra dimensions is a general feature in theories beyond the standard model especially in theories for unifying gravity and other forces, such as superstring theory. These extra dimensions should be "hidden" for consistency with observations. Various scenarios for "hidden" extra dimensions have been proposed for example, a Brane world with large compact extra dimensions in factorizable geometry proposed by Arkani- Hamed et al. [1], a Brane world with non compact extra dimensions in nonfactorizable geometry proposed by Randall and Sundarum (1999) [6, 7]. In this chapter, we employ the simplest scenario : small compact extra dimensions in factorizable geometry, as introduce in Kaluza – Klein theories (1926) [4, 5].

Several workers have recently obtained exact solutions using higher dimensional space-time for both cosmological and non cosmological cases, with or without matter. Hajj and Boutors (1991) [3] obtained solutions for an LRS Bianchi model, with a time dependent equation of state p = f(t). But their solution have been shown to be untenable for the entire range of the cosmic time, because of inappropriate scale transformation adopted by Josef Hajj- Boutros in his formation (Banerjee et al.1990)[2].

This work is generalization of the work obtained earlier by Manna and Bhui (1994). In this work we have obtained a homogeneous cosmological model in higher dimensions by assuming time dependent equation of $\Lambda = \alpha \frac{\dot{R}^2}{R^2}$ in the content of Kaluza Klein theory. The solution obtained in this work is realistic in the sence that physical properties, such as mass, density remain positive throughout the entire evaluation of the universe. The dynamical behavior of the model is examine and it is noted that with a decrease in extra space the observable 3D space entropy increases, thus accounting for large value of entropy observable at present.

II. EINSTEIN FIELD EQUATIONS

Consider the five dimensional line element of the form:

$$ds^2 = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2) - A^2(t)dm^2, \quad (1)$$

where spatial curvature is assumed to be zero and the metric coefficients to be functions of time only.
The energy - momentum tensor for a perfect fluid, in general, is given by

$$T_{ab} = (\rho + p)\,\mu_a \mu_b - pg_{ab} \quad (2)$$

where v^μ is the velocity in the 5th dimensions, ρ and p are the matter's density and isotropic pressure.

We use a co-moving co-ordinate system $v_\mu = \delta_0^\mu$, and the Einstein field equations can be written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} + \Lambda g_{\mu\upsilon}, \quad (3)$$

with the help of $\Lambda = \alpha \frac{\dot{R}^2}{R^2}$ the Einstein field equations by using (1) - (3) can be expressed as

$$(3-\alpha)\frac{\dot{R}^2}{R^2}+3\frac{\dot{R}\dot{A}}{RA}=\rho, \tag{4}$$

$$\frac{2\ddot{R}}{R}+(1-\alpha)\frac{\dot{R}^2}{R^2}+\frac{2\dot{R}\dot{A}}{RA}+\frac{\ddot{A}}{A}=-p, \tag{5}$$

$$\frac{3\ddot{R}}{R}+(3-\alpha)\frac{\dot{R}^2}{R^2}=-p_5. \tag{6}$$

Here the dot over the variables represents derivatives with respect to time.
We assume that here is no excitation in the extra space, so that $p_5=0$.
Thus Equation (6) can be written as

$$\frac{3\ddot{R}}{R}+(3-\alpha)\frac{\dot{R}^2}{R^2}=0. \tag{7}$$

Again from Equation (7) we get the shear factor R as,

$$R=(K_1t+K_2)^{\frac{3}{6-\alpha}}, \tag{8}$$

where K_1 and K_2 are arbitrary constants of integration.
As mentioned earlier, we assume an equation of state $p = f(t)\rho$,
we get from the Eqns. (4) and (5) the following two modified equations:

$$f\ (3-\alpha)\frac{\dot{R}^2}{R^2}+3f\frac{\dot{R}\dot{A}}{RA}=p, \tag{9}$$

$$(1-\alpha)\frac{\dot{R}^2}{R^2}+2\frac{\dot{R}\dot{A}}{RA}+\frac{2\ddot{R}}{R}+\frac{\ddot{A}}{A}=-p. \tag{10}$$

Eqns. (9) and (10) on simplification, yield the differential equation as follows

$$(f\alpha+3f+1+\alpha)\frac{\dot{R}^2}{R^2}+(2+3f)\frac{\dot{R}\dot{A}}{RA}+\frac{2\ddot{R}}{R}+\frac{\ddot{A}}{A}=0. \tag{11}$$

A time translation in Equation (8) will make $K_2 = 0$ hence

$$R=(K_1t)^{\frac{3}{6-\alpha}}. \tag{12}$$

Using the value of R from Eqn. (12) in (11) we get

$$t^2\ddot{A}+a_1(3f+2)t\dot{A}+a_1^2(3f-\alpha f+a_2)A=0, \tag{13}$$

where $a_1=\dfrac{3}{6-\alpha}$ and $a_2=1-\alpha+\dfrac{2}{3(\alpha-3)}$.

The above equation can also be written as

$$p_0\ddot{A}+p_1\dot{A}+p_2A=0, \tag{14}$$

Where

$$p_0=t^2, \tag{15a}$$

$$p_1=a_1t(2+3f), \tag{15b}$$

$$p_2=a_1^2(3f-f\alpha+a_2). \tag{15c}$$

Equation (14) is exact if $p_2-\dot{p}_1+\ddot{p}_0=0.$ (16)

Using values from equation (15) in (16) we get

$$f=\frac{a_4}{a_3}-at^{a_3}, \tag{17}$$

where $a_3 = \dfrac{3a_1^{\,2} - aa_1^{\,2} - 3a_1}{3a_1}$ and $a_4 = \dfrac{a_1^{\,2} a_2 - 2a_1 + 2}{3a_1}$.

Putting the value of $f(t)$ from Equation (16) in Equation (13), we get

$$t^2 \ddot{A} + \left(a_5 - 3aa_1 t^{a_3}\right) t \dot{A} + \left(a_6 - aa_1^{\,2}(3-\alpha) t^{a_3}\right) A = 0, \tag{18}$$

where $a_5 = -\dfrac{3a_4 a_1}{a_3} + 2a_1$ and $a_6 = -\dfrac{3a_4 a_1^{\,2}}{a_2} + \dfrac{\alpha}{a_2} a_4 a_1^{\,2} + a_2 a_1^2$.

Equation (17) is exact differential Equation while first integral is given by

$p_0 \dot{A} + (p_1 - \dot{p}_0) A = C_2$,

therefore, we get

$$t^2 \dot{A} + t\left(a_7 - 3aa_1 t^{a_3}\right) A = C_2, \tag{19}$$

where $a_7 = 2a_1 - 2 - \dfrac{3a_1 a_4}{a_3}$ and C_2 is an arbitrary integration constant.

A straightforward calculation of Equation (18) gives

$$A = C_3 \exp\left\{\frac{3a_1 a t^{a_3}}{a_3}\right\} t^{-a_7} - \frac{C_2}{3a_1 a} t^{-a_7}, \tag{20}$$

where C_3 is another integration constant.

This is a new solution for a perfect fluid in higher dimensional space - time. It is encouraging to note that as $t \to \infty$, $A \to 0$. So our space - time shows the desired property of dimensional reduction.

After substituting the values of R and Λ in the field equations from (8) and (9) we get the expressions for ρ and p as follows

$$\rho = \frac{-54}{(6-\alpha)^2 t^2} + \frac{27aa_1}{(6-\alpha) t^{2-a_3}} \left[\frac{1}{1 - \dfrac{C_2}{3C_3 aa_1 \exp\left\{\dfrac{3aa_1 t^{a_3}}{a_3}\right\}}} \right], \tag{21}$$

and

$$p = \frac{(36 - a_8)}{(6-\alpha)^2 t^2} - \frac{9aa_1}{t^{2-a_3}} \left[\frac{\dfrac{2}{6-\alpha} - \dfrac{a_5}{2} + aa_1 t^{a_3}}{1 - \dfrac{C_2}{3C_3 aa_1 \exp\left\{\dfrac{3aa_1 t^{a_3}}{a_3}\right\}}} \right], \tag{22}$$

where $a_8 = 9\alpha - 27 + (6-\alpha)^2 a_7 a_5 - a_6 (6-\alpha)^2$.

So the constant a is related to the total mass of the system; when a = 0, the mass density gives $\rho = \dfrac{-54}{(6-\alpha)^2 t^2}$ and pressure is $p = \dfrac{(36 - a_8)}{(6-\alpha)^2 t^2}$. As $t \to 0$, ρ and $p \to \infty$ and as $t \to \infty$, ρ and $p \to 0$.

Before concluding this session let us very briefly discuss the dynamical behavior of the model. The four-volume $V = R^3 A$ starts from zero at t = 0 and at $t_0 = \dfrac{9a^2}{[(1-\alpha)\ln \frac{C_1}{6aC_2}]^2}$.

It is interesting to note that the extra scale factor, starting from an infinite extension at the big bang, reduce to the Planckian length at the same point of time.

III. CONCLUSION

We are presented hear the solution of five dimensional Kaluza-Klein type metric by assuming the cosmological constant Λ of the form $\Lambda = \alpha \dfrac{\dot{R}^2}{R^2}$ with energy momentum tensor containing perfect fluid. From the solution of metric coefficient A, it follows that over time A approaches to zero, giving rise to the phenomenon of a dimensional reduction for the expanding model. It is also observed that from the expression for mass density and pressure, they are, vanishes at $t \to \infty$ and infinite at $t \to 0$.

Since $n \propto R^{-3}$ the effective 4- dimensional specific entropy will in general increase at a rate (Banergee et al., 1990)

$$\dot{\sigma}_4 = \frac{1}{kT}\left[\frac{d}{dt}\left(\frac{\rho}{n}\right) + p\frac{d}{dt}\left(\frac{1}{n}\right)\right]$$

$$\dot{\sigma}_4 = \frac{1}{nkt}\left[\dot{\rho} + (\rho + p)3\frac{\dot{R}}{R}\right],$$

$$\dot{\sigma}_4 = \frac{1}{nkT}\left(-\rho\frac{\dot{A}}{A}\right). \qquad (23)$$

From equation (23), we thus arrive at a very important relation, which needs further interpretation in higher dimensional physics.

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