Mechanisms of 10 Links, 13-Joints, 1-F Kinematic Chains of Group B

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Abstract

Author’s objective is to develop a new, easy, reliable, and efficient method to detect isomorphism and prepare a catalogue of fixed link and its corresponding equivalent links in the distinct mechanisms in kinematic chains of Group B. It will help the new researchers / designers to select the best mechanism kinematic chain and mechanism to perform the desired task at the conceptual stage of design. The proposed method is presented by comparing the structural invariants ‘sum of the absolute values of the characteristic polynomial coefficients’ [SCPC] and ‘maximum absolute value of the characteristic polynomial coefficient’ [MCPC] of [JJ] matrices. These invariants may be used to detect isomorphism in the mechanism kinematic chain having simple joints. The method is explained with the help of examples of planar kinematic chain having simple joints.

Keywords: Kinematic Chain; Fixed Link; Equivalent Link; SCPC; MCPC

NOTATIONS USED

F.L.-Fixed Link, E.L.-Equivalent Link, n5 n4 n3 n2 –Pentagonal, Quaternary, Ternary, Binary Links

I. INTRODUCTION

Over the past several years much work has been reported in the literature on the structural synthesis of kinematic chains and mechanisms. Undetected isomorphism results in duplicate solutions and unnecessary effort. Therefore, the need for a reliable and efficient algebraic method for this purpose is necessary. Identifying isomorphism among kinematic chains using characteristic polynomials of adjacency matrices of corresponding kinematic chains are simple methods [Uicher and Raicu 1975, Mruthyunjaya and Raghavan 1979, Yan and Hall, 1981]. But the reliability of these methods was in question as several counter examples were found by Mruthyunjaya [Mruthyunjaya, 1987]. The test proposed by Mruthyunjaya [Mruthyunjaya, 1987] is based on characteristic coefficients of the ‘Degree matrix’ of the graph of the kinematic chains. The elements of the degree matrix were sum of the degree of vertices (degree or type of links) or unity in a link-link adjacency matrix. Later on this test was also found unreliable. Krishnamurthy [Mruthyunjaya, 1987] proposed the representation polynomial for detecting isomorphism between two kinematic chains. The representation polynomial is the determinant of the generalized adjacency matrix, called representation matrix of the kinematic chain. But the representation matrix requires the use of a large number of symbols, the calculation and comparison of the representation polynomials is not as easy as that of the characteristic coefficients of the adjacency matrix. Balasubramanian and Parthasarthy [Balasubramanian and Parthasarthy 1981] proposed the procedure based on the concept of the permanent of a matrix for the purpose. Tang and Liu [Tang, Liu, Tyng, 1993] presented a method based on degree code as mechanism identifier. Several other methods like comments [Cubillo and Wan, Jinbao, 2005], [Hasan A., 2007, 2009, 2010, 2012] are also in use.

II. THE JOINT-JOINT [JJ] MATRIX

This matrix is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size n x n, where n is the number of joints in a kinematic chain.

\[ [JJ] = \begin{bmatrix} L_{ij} \end{bmatrix}_{n \times n} \quad \text{------- (1)} \]

Where

\[ L_{ij} = \begin{cases} \text{Degree of link between } i^{th} \text{ and } j^{th} \text{ joints} \\ \text{those are directly connected} \\ =0, \text{ if joint } i \text{ is not directly connected to joint } j \end{cases} \]

Off course all the diagonal elements \( L_{ii} = 0 \)
III. CHARACTERISTIC POLYNOMIAL OF [JJ] MATRIX

D (λ) gives the characteristic polynomial of [JJ] matrix. The monic polynomial of degree n is given by equation (2).

\[ | (JJ - \lambda I) | = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots + a_n \], \lambda + a_n \]

(2)

Where; n = number of simple joints in kinematic chain and
1, a_1, a_2, a_{n-1}, a_n are the characteristic polynomial coefficients.

The two important properties of the characteristic polynomials are

A. The sum of the absolute values of the characteristic polynomial coefficients (SCPC) is an invariant for a [JJ] matrix. i.e.

\[ | 1 | + | a_1 | + | a_2 | + \cdots + | a_{n-1} | + | a_n | = \text{invariant} \]

B. The maximum absolute value of the characteristic polynomial coefficient (MCPC) is another invariant for a [JJ] matrix.

IV. STRUCTURAL INVARIANTS [SCPC] AND [MCPC]

The values of characteristic polynomial coefficients are invariants for a [JJ] matrix. To make these [JJ] matrix characteristic polynomial coefficients as a powerful single number characteristic index, new composite invariants have been proposed. These invariants are ‘SCPC’ and ‘MCPC’. These invariants are unique for a [JJ] matrix and may be used as identification numbers to detect the isomorphism among simple jointed kinematic chains. The characteristic polynomial coefficients values are the characteristic invariants for the kinematic chains. Many investigators have reported co-spectral graph (non-isomorphic graph having same Eigen spectrum). But these Eigen spectra (Eigen values or characteristic polynomial) have been determined from (0, 1) adjacency matrices. The proposed [JJ] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chains having co-spectral graphs. Therefore, it is verified that the structural invariants ‘SCPC’ and ‘MCPC’ are capable of characterizing all kinematic chains and mechanisms uniquely. Hence, it is possible to detect isomorphism among all the given kinematic chains.

V. ISOMORPHISM OF KINEMATIC CHAINS

1) Theorem: Two similar square symmetric matrices have the same characteristic polynomials.

2) Proof: Let the two kinematic chains are represented by the two similar matrices A and B such that B = P^{-1} AP, taking into account that the matrix λI commutes with the matrix P and | P^{-1} | = | P |^{-1}. Since the determinant of the product of two square matrices equals the product of their determinants, we have

\[ | B - \lambda I | = | P^{-1} A P - \lambda I | \]

\[ = | P^{-1} (A - \lambda I) P | \]

\[ = | P^{-1} | | (A - \lambda I) | | P | = | A - \lambda I | \]

Hence, D (λ) of ‘A’ matrix = D (λ) of ‘B’ matrix.

D (λ) = characteristic polynomial of the matrix.

It means that if D (λ) of two [JJ] matrices representing two kinematic chains is same, their structural invariants ‘SCPC’ and ‘MCPC’ will also be same and the two kinematic chains are isomorphic otherwise non-isomorphic chains.

VI. ILLUSTRATIVE EXAMPLE 1 (IDENTIFICATION OF CO-SPECTRAL GRAPHS)

The non-isomorphic kinematic chains have the same characteristic polynomials using (0, 1) adjacency matrices and their kinematic graphs are called as Co-spectral graphs. But the characteristic polynomials of such chains derived from [JJ] matrices are distinct. Therefore, the structural invariants [SCPC] and [MCPC] are also distinct. This is proved with the help of examples of two kinematic chains with 10 bars, 12 joints, three degree of freedom as shown in Fig 1. The task is to examine whether these two chains are isomorphic. The structural invariants of these two chains are as follows:

- For chain 2(a): [SCPC] = 8.3734e+006 , [MCPC] =3.5938e+006
- For chain 2(b): [SCPC] = 7.0147e+006 , [MCPC] = 2.9393e+006

Our method reports that chain 2(a) and 2(b) are non-isomorphic as the set of values of [SCPC] and [MCPC] are different for both the kinematic chains. Note that by using other method comments [Cubillo and Wan, Jinbao, 2005], the same conclusion is obtained.
VII. RESULTS

The proposed invariants [SCPC] and [MCPC] are used as the identification number of the kinematic chains having simple joints. The identification numbers of all 1-dof kinematic chains up to 10-Links are with the author. These invariants are also able to detect isomorphism among the kinematic chains with multiple joints also. Fixed Links and Equivalent Links in Distinct Mechanisms of 10 Links, 13 Joints, Single degree of freedom Kinematic Chains Group B are listed in Table 1(B).

VIII. CONCLUSIONS

In this paper, a simple, efficient, and reliable method to identify isomorphism is proposed. By this method, the isomorphism of mechanisms kinematic chains can easily be identified. It incorporates all features of the kinematic chains and as such, violation of the isomorphism test is rather difficult. The method has been found to be successful in distinguishing all known 16 kinematic chain of 8-links, 230 kinematic chain of 10-links having 1-F. The advantage is that they are very easy to compute using MATLAB software. It is not essential to determine both the structural invariants to compare two chains, only in case the [SCPC] is same then it is needed to determine [MCPC] for both kinematic chains. The [JJ] matrices can be written with very little effort, even by mere inspection of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom.

Table - 1(B)

<table>
<thead>
<tr>
<th>Kinematic Chain</th>
<th>n₁n₂n₃n₄</th>
<th>EZDV</th>
<th>F. L.</th>
<th>E. L.</th>
<th>D. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1036</td>
<td>4030</td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>SCPC(B1) = 2.9434e+007</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCPC(B1) = 1.2179e+007</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1036</td>
<td>3220</td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>SCPC(B2) = 8.3443e+007</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCPC(B2) = 3.0804e+007</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Mechanics of 10 Links, 13-Joints, 1-F Kinematic Chains of Group B

<table>
<thead>
<tr>
<th>Kinematic Chain</th>
<th>$n_1n_2n_3n_4$</th>
<th>EVD</th>
<th>F. L.</th>
<th>E. l.</th>
<th>D. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{B3}$</td>
<td>1036 3220</td>
<td>a b c e g i</td>
<td></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>$\text{B3}$</td>
<td>1036 3220</td>
<td>a  b  c  d</td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>$\text{B4}$</td>
<td>1036 3220</td>
<td>a  b  c  d</td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>$\text{B5}$</td>
<td>1036 3220</td>
<td>a  b  c  d</td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>$\text{B6}$</td>
<td>1036 3220</td>
<td>a  b  c  d</td>
<td></td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

**Table 1(B) continued.**

$SCPC(B3) = 1.0459e+008$
$MCPC(B3) = 3.6124e+007$

$SCPC(B4) = 7.5611e+007$
$MCPC(B4) = 2.2178e+007$

$SCPC(B5) = 9.8280e+007$
$MCPC(B5) = 2.9778e+007$

$SCPC(B6) = 1.3308e+008$
$MCPC(B6) = 4.4750e+007$
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<table>
<thead>
<tr>
<th>Kinematic Chain</th>
<th>n₁n₂n₃</th>
<th>EZDV</th>
<th>F. L.</th>
<th>E. l.</th>
<th>D. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1036 2410</td>
<td>a b c d e f g h i j</td>
<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
</tbody>
</table>

SCPC(B7) = 9.3750e+007
MCPC(B7) = 3.0903e+007

Table - 1(B) continued.

<table>
<thead>
<tr>
<th>Kinematic Chain</th>
<th>n₁n₂n₃</th>
<th>EZDV</th>
<th>F. L.</th>
<th>E. l.</th>
<th>D. M.</th>
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</thead>
<tbody>
<tr>
<td>1036 2410</td>
<td>a b c d e f g h i j</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>

SCPC(B8) = 1.3743e+008
MCPC(B8) = 5.2656e+007

<table>
<thead>
<tr>
<th>Kinematic Chain</th>
<th>n₁n₂n₃</th>
<th>EZDV</th>
<th>F. L.</th>
<th>E. l.</th>
<th>D. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1036 2410</td>
<td>a b c d e f g h i j</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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</tbody>
</table>

SCPC(B9) = 3.5777e+008
MCPC(B9) = 1.4640e+008

<table>
<thead>
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<th>Kinematic Chain</th>
<th>n₁n₂n₃</th>
<th>EZDV</th>
<th>F. L.</th>
<th>E. l.</th>
<th>D. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1036 2410</td>
<td>a b c d e f g h i j</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

SCPC(B10) = 3.1304e+008
MCPC(B10) = 1.0261e+008

<table>
<thead>
<tr>
<th>Kinematic Chain</th>
<th>n₁n₂n₃</th>
<th>EZDV</th>
<th>F. L.</th>
<th>E. l.</th>
<th>D. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1036 2410</td>
<td>a b c d e f g h i j</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

SCPC(B11) = 1.6131e+008
MCPC(B11) = 4.9551e+007
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(IJSTE/ Volume 2 / Issue 08 / 027)

SCPC(B12) = 3.1755e+008
MCPC(B12) = 8.5510e+007

Table - 1(B) continued.

<table>
<thead>
<tr>
<th>Kinematic Chain</th>
<th>n5 n4 n3 n2</th>
<th>EZDV</th>
<th>F. L.</th>
<th>E. l.</th>
<th>D. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1036 2410</td>
<td>a b c d e f g h i j</td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SCPC(B13) = 2.0396e+008
MCPC(B13) = 8.6620e+007

SCPC(B14) = 3.4929e+008
MCPC(B14) = 1.3072e+008

SCPC(B15) = 6.7910e+008
MCPC(B15) = 3.0248e+008

TOTAL NUMBER OF DISTINCT MECHANISMS OF GROUP B = 126

REFERENCES


