Finite Element Analysis of Bicycle Crank

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Abstract

An attempt has been made to analyze the crank of a bicycle to check its structural integrity under the operating condition. Finite Element Method is used as a tool for this purpose. The elements used for this analysis are 3-D Beam and 10-node Tetrahedron (SOLID 92). The crank is analyzed in static condition. Distribution of different stress components and the maximum von mises stress have been ascertained. It has been found that the maximum von mises stress in the crank is 218.226 MPa which is below the yield strength of the crank material (245 Mpa).

Keywords: Tetrahedron (SOLID 92), static condition, von mises stress

I. INTRODUCTION

A. Introduction to Fem

Finite element analysis (FEA) is a fairly recent discipline crossing the boundaries of mathematics, physics, engineering and computer science. The method has wide application and enjoys extensive utilization in the structural, thermal and fluid analysis areas.

The finite element method (FEM) is a numerical technique for finding approximate solutions of partial differential equations (PDE) as well as integral equations and when this method is used to analyze different physical problems it is known as Finite Element Analysis (FEA).

The Finite Element Method has roots in many disciplines, the end result is a technology that is so advanced that it is indistinguishable from magic. The vast catalog of capability that comprises FEA will no doubt grow considerably large in future. CAE is here to stay, but in order to harness its true, the user must be familiar with many concepts, including mechanics of the problem being modeled. All analysis requires time, experience and most importantly careful planning.

II. GEOMETRY OF THE CRANK

A. Geometry of the Crank

Geometry of the crank was obtained by direct measurement using vernier calipers and radius gauge. Part drawing of the crank was prepared using Solid Edge and is shown in fig. 2.1
2.2 Composition and mechanical properties of the crank: Chemical composition of the material of the crank was obtained using Spectro spark emission test and following was found to be its composition:

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Element</th>
<th>Percentage</th>
<th>Sl No.</th>
<th>Element</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Carbon</td>
<td>0.244</td>
<td>7</td>
<td>Nickel</td>
<td>0.066</td>
</tr>
<tr>
<td>2</td>
<td>Silicon</td>
<td>0.139</td>
<td>8</td>
<td>Molybdenum</td>
<td>0.016</td>
</tr>
<tr>
<td>3</td>
<td>Manganese</td>
<td>0.520</td>
<td>9</td>
<td>Copper</td>
<td>0.109</td>
</tr>
<tr>
<td>4</td>
<td>Phosphorus</td>
<td>0.036</td>
<td>10</td>
<td>Vanadium</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>Sulphur</td>
<td>0.054</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Chromium</td>
<td>0.065</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This indicates that the material is typical C25 steel and following are its properties:

<table>
<thead>
<tr>
<th>Properties</th>
<th>Specification</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>ASTM A216</td>
<td>WCB</td>
</tr>
<tr>
<td>Property (C25</td>
<td>(American</td>
<td>specification of carbon steel is suitable for</td>
</tr>
<tr>
<td>Steel)</td>
<td>Society for Testing and Materials) A216</td>
<td>fusion welding at high temperatures. WCB stands for</td>
</tr>
<tr>
<td></td>
<td>specification of carbon steel is suitable for</td>
<td>“Wrought Carbon” for grade B. In grade A, B &amp; C</td>
</tr>
<tr>
<td></td>
<td>fusion welding at high temperatures. WCB stands for</td>
<td>grade B has good ductility and tensile strength as</td>
</tr>
<tr>
<td>Mechanical</td>
<td>grade B has</td>
<td>compared to grade A &amp; C.</td>
</tr>
<tr>
<td>Properties</td>
<td>good ductility and tensile strength as compared to</td>
<td>grade A &amp; C.</td>
</tr>
<tr>
<td>Tensile</td>
<td>485 Mpa</td>
<td></td>
</tr>
<tr>
<td>Strength</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Strength</td>
<td>245 Mpa</td>
<td></td>
</tr>
<tr>
<td>Shear Strength</td>
<td>150 Mpa</td>
<td></td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

### B. Solid Modeling:

Solid model of the crank has been prepared in solid edge.

The solid model of the crank has also been built in ANSYS and following were the Commands used Key points, Arc, Volume, Lines, and Areas.
C. Load Calculations:

1) Calculation of ground reactions

![Free Body Diagram of a Crank Shaft](image)

Taking Moments about A
\[ F_b \times 1100 + F \times 800 + F_c \times 550 = F_1 \times 1100 \]
\[ 735.75 \times 1100 + 784.8 \times 800 + 178.542 \times 550 = F_1 \times 1100 \]
\[ F_1 = 1395.78 \text{ N} \]

Taking Moments about B
\[ F_1 \times 300 + F_c \times 550 = F_2 \times 1100 \]
\[ 784.8 \times 300 + 178.542 \times 550 = F_2 \times 1100 \]
\[ F_2 = 303.307 \text{ N} \]

Now, Frictional force
\[ F_f = \mu F_1 + \mu F_2 \quad (\mu = \text{Co-efficient of friction}) \]
\[ = 0.1 \times 1395.78 + 0.1 \times 303.307 = 169.90 \text{ N} \]

2) Calculation of pedaling force:

![Force Body Diagram of Crank mechanism](image)
Z = No. of teeth; F_f = Frictional force; F_p = Pedaling Force

Radius of Back Wheel = R = 320 mm
Radius of Smaller Sprocket = r_1 = 36.8 mm
Radius of Spider Sprocket = r_2 = 90 mm
No. of teeth on Smaller Sprocket = Z_1 = 18
No. of teeth on Bigger Sprocket = Z_2 = 44
Teeth Ratio = Z_2/Z_1 = 44/18 = 2.44

Weight of the bicycle = 18.2 kg
Weight of two persons sitting on it = 80 + 75 = 155 kg.

For equilibrium of the wheel,

\[ F_f \times 320 = F_3 \times 36.8 \]
\[ 169.90 \times 320 = F_3 \times 36.8 \]
\[ F_3 = 1477.391 \, N \]

Now, Teeth Ratio = G = Z_2/Z_1 = T_4/T_3
\[ \gg 2.44 = T_4/(1477.391 \times 36.8) \]
\[ \gg T_4 = 132657.892 \, N \]

But,

\[ T_4 = \text{Pedaling Force (F_p) \times Crank Length} \]
\[ \gg 132657.892 = F_p \times 179 \]

Pedaling Force = F_p = 741.10 N \[ \cong 750 \, N \]

III. THEORETICAL ANALYSIS OF THE CRANK

A. LOADS ON THE CRANK

To build confidence in the analysis, approximate value of maximum stress is determined using theory of bending using mean cross section of the crank.

B. STRESS CALCULATIONS:

The bending equation is given by

\[ \frac{M}{I} = \frac{F}{y} \]

Where M = Bending Moment = F.L
F = Force Applied, L = Distance from the fixed end

\[ I = \text{Moment of inertia} = \frac{bd^3}{12} \]

Where b = breadth; d = depth
y = Distance from the neutral plane;
f=Max bending stress
In the fig shown the load is being applied eccentrically. This eccentric load can be converted into direct load and a Torsional moment as shown in Fig below

\[ T = 750 \times 55 = 41250 \text{ N-mm} \]
Here, \[ M = F \cdot L = 750 \times 179 = 134250 \text{ N-mm} \]
\[ I = \frac{bd^3}{12} = \frac{13.88 \times 16.33^3}{5036.93} \approx 5036.93 \text{ mm}^4 \]
\[ y = \frac{d}{2} = \frac{16.33}{2} = 8.165 \text{ mm} \]
Now substituting the above values in equation 1 we get
\[ f = \frac{M}{I} = \frac{134250}{5036.93} = 26.622 \text{ N/mm}^2 \]
Max Bending Stress = \( f = 217.622 \text{ N/mm}^2 \)

To find the torsional stress in the beam we have the formula
\[ \tau = \frac{T}{kab^2} \]
Where 
- \( T \) – Torsional moment
- \( k \) – constant (refer table
- \( a \) – breadth
- \( b \) – height

<table>
<thead>
<tr>
<th>( a/b )</th>
<th>( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.197</td>
</tr>
<tr>
<td>1</td>
<td>0.208</td>
</tr>
<tr>
<td>1.2</td>
<td>0.231</td>
</tr>
<tr>
<td>1.5</td>
<td>0.246</td>
</tr>
</tbody>
</table>

\[ \tau = \frac{750 \times 55}{0.197 \times 13.88 \times 16.33^2} = 56.571 \text{ N/mm}^2 \]

To find 1st principal stress:
\[ \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left\{\left(\sigma_x - \sigma_y\right)/2 + \tau^2\right\}^2} \]
\[ \sigma_y = 0 \quad \text{(Y-component stress is zero in 1-D analysis)} \]
\[ \sigma_x = 217.622 \text{ N/mm}^2 \]
\[ \sigma_1 = \frac{217.622}{2} + \sqrt{\left\{\left(\frac{217.622}{2}\right)^2 + \tau^2\right\}^2} \]
\[ \sigma_1 = 231.45 \text{ N/mm}^2 \]
Von mises stress:
\[ \sigma_v = \sqrt{(\sigma_1^2 - \sigma_1 \sigma_x + \sigma_x^2 + 3 \tau^2)}^{1/2} \]
\[ \sigma_v = (231.45^2 + 3 \times 56.571)^{1/2} \]
\[ \sigma_v = 251.33 \text{ N/mm}^2 \]

IV. 2-D Analysis in ANSYS

A. 2-D Model of the crank:
As a second step towards confidence building in the analysis approximate analysis has been carried out using 1-D space beam element BEAM 4
The crank model with boundary conditions applied is shown in fig 4.2.

**B. Analysis Details:**

1) Element type: Beam 4
2) Real constants:
3) Material Properties:
   1) Young’s Modulus: $2 \times 10^5$ N/mm$^2$
   2) Poisson’s Ratio: 0.3
4) Mesh Size (Element Edge Length): 10 mm

C. Results of 2-D Analysis:

1) Maximum Displacement in the crank = 2.63 mm (At the free end of the crank)
2) Maximum Stress produced in the crank = 217.533 N/mm² (At the fixed end of the crank)

The Plots of displacement and stresses are shown below

Fig. 4.3: Set 2 (Circular Cross section)

Fig. 4.5: Plot of displacement in the crank
It can be seen that maximum stress value obtained by hand calculations in section 3.2 is equal to that obtained in ANSYS and hence the analysis carried out in ANSYS is correct.

V. 3-D ANALYSIS

A. Meshing Details

The element that has been used for this analysis is 3-D 10 node tetrahedral structural Solid (SOLID 92). The geometry of the element has been shown in the fig below

It has a quadratic displacement behavior and is well suited to model irregular meshes (such as produced from various CAD/CAM systems). The element is defined by ten nodes having three degrees of freedom at each node: translations in the nodal x, y and z directions. The element also has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. Pressures may be input as surface loads on the element faces.

Mesh details for the converged results
1) No. of nodes created = 50944
2) No. of elements created = 33424

The meshing is done on the model by using mesh tool option and mesh size 4.5 and it is Shown in the fig 5.2.
B. Imposition of Loads and Constraints

Imposition of constraints:
For performing the analysis the model has to be constrained and this is shown in fig 5.3.

Constraint details:
Constrained Degrees of Freedom: ALL DOF
Entities being constrained: Nodes
No. of Constrained nodes: 5127 out of 50944
Note: The area being constrained is a hole through which a carriage bolt goes in.

C. Load imposition:
Note: The load that has been calculated in section 2.4.2 is applied on the model in negative Y-direction.

Load Details:
Entities on which the loads have been applied: Nodes
No of nodes on which loads have been applied: 40

\[
\text{Load applied on each node:} \quad \frac{\text{Total Load}}{\text{No of nodes}} = \frac{750}{420} = 1.785 \text{ N}
\]

VI. ANALYSIS AND RESULTS

D. Stress Distribution in the Crank:
The stress distributions obtained using ANSYS are shown in fig 6.1-6.4

Fig. 6.1 X-Component Stress distribution in the crank

Fig. 6.2 Shear Stress distribution in XY Plane

Fig. 6.3 1st Principal Stress distributions
The variation of stresses for different mesh sizes are noted down and a graph of stress vs mesh size is plotted.

![Von Mises Stress Distribution](image)

Fig. 6.4 Von Mises Stress Distribution

Comparison of theoretical results with ANSYS results are shown in table below

<table>
<thead>
<tr>
<th>Maximum Stresses</th>
<th>Theoretical</th>
<th>ANSYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Component Stress</td>
<td>217.622</td>
<td>212.792</td>
</tr>
<tr>
<td>1st Principal Stress</td>
<td>231.45</td>
<td>220.446</td>
</tr>
<tr>
<td>Shear Stress</td>
<td>56.571</td>
<td>104.335</td>
</tr>
<tr>
<td>Von Mises</td>
<td>251.33</td>
<td>218.226</td>
</tr>
</tbody>
</table>

VIII. CONCLUSION

Following conclusions may be drawn based on the analysis carried out during this project:

1) The Maximum von mises stress induced in the crank is 218.226(Mpa) which is well below the ultimate tensile strength of the material(485Mpa) which gives us a FOS of 2.22 and therefore the crank does not fail.

2) The yield strength of the material is 245 Mpa which is below the Von mises stress (218.226 Mpa).

3) The Shear stress induced in the material (104.335 Mpa) is also well below the shear strength of the material (150 Mpa) hence the crank is safe under torsion.

4) The crank is structurally sound under operating conditions.
REFERENCES