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Green's Function of Optical Systems with Amplitude Apodization Filters

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Abstract

The Green's function of an optical system with circular apertures using rotationally symmetric amplitude apodisation filters has been investigated. The elegant principles of Fourier Transform Optics have been used for this purpose. The results obtained by us have been presented in the form of figures and tables. Finally, the important conclusions that can be drawn from our present studies have been mentioned.

Keywords: Mathematical Optics, Green's Function, Apodization, etc

I. INTRODUCTION

A large number of investigations have been carried out on the distributions of light at and near the focal plane and thus, certain general features of the diffracted field near and far away from the Gaussian focal plane due to perfect systems have been established [1]. All the analytical investigations are based on the Impulse Response of the optical system to the intensity of a point source of light. This Intensity Impulse Response is known as the Green's Function in Cosmology, Field Theory and Mathematical Optics [2]. In the present paper, we have studied the Intensity Impulse Response (IIR) of an apodized optical system. The results obtained by us have been discussed with relevant figures and tables.

II. ANALYTICAL EXPRESSION FOR GREEN'S FUNCTION

The diffracted light amplitude at a point away from the Gaussian focal plane is given by [3].

$$G(y,z) = 2\int_0^1 f(r)\exp\left(\frac{-iyr^2}{2}\right) J_0(zr)\, r\, dr \qquad \ldots \quad (1)$$

Where f(r) is the pupil function of the system under consideration; y and z are the usual diffraction variables in the axial and transverse directions respectively; $J_0(zr)$ is the Bessel function of the first kind and the zero order; r is the radial co-ordinate of an arbitrary point on the pupil.

In the present study, we have considered a circularly symmetric apodized system whose pupil function can be analytically expressed as:

$$f(r) = \frac{1+\beta\cos\pi r^2}{1+\beta} \qquad \ldots \quad (2)$$

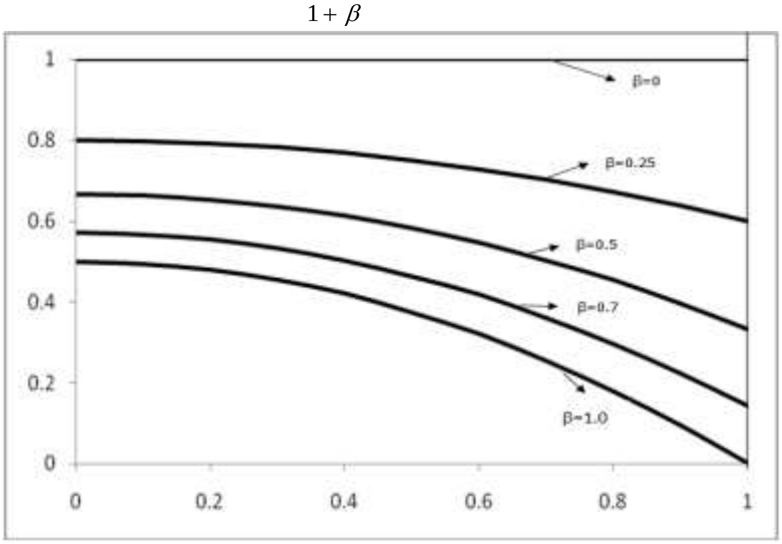

Fig. 1: Pupil Transmission curves for various values of β.

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where β is the apodisation parameter which controls the degree of non-uniformity of the transmitted beam. The pupil transmission curves for various values of β have been shown in the figure 1.
Substituting equation (2) in (1), we get

$$G(y,z) = 2\int_0^1 \frac{1+\beta\cos\pi r^2}{1+\beta}\ \exp\left(\frac{-iyr^2}{2}\right)\ J_0(\mathrm{zr})\ \mathrm{r\ dr} \quad \ldots \quad (3)$$

The diffracted amplitude at the Gaussian focal plane (y = 0) is given by:

$$G(0,z) = 2\int_0^1 \frac{1+\beta\cos\pi r^2}{1+\beta}\ J_0(\mathrm{zr})\ \mathrm{r\ dr} \quad \ldots \quad (4)$$

For a clear aperture, i.e., for $\frac{1+\beta\cos\pi r^2}{1+\beta}$ = 1, we can write

$$G(0,z) = 2\int_0^1 J_0(\mathrm{zr})\ \mathrm{r\ dr} \quad \ldots \quad (5)$$

The quantity $G(y,z)$ in equation (3) is known as the Amplitude Point Spread Function or the Amplitude Green's Function of the system. The amplitude Green's function is not a detectable quantity as it produces a fastly varying oscillating field and therefore, it cannot be detected and measured. It is the Intensity Point Spread or Green's Function which produces a stationary field can be detected or measured. It this Intensity PSF which represents the light energy falling or passing through the optical system. The intensity PSF is given by the squared modulus of the amplitude PSF. Thus, from the equation (4), we get:

$$I(0,z) = \left[\ 2\int_0^1 \frac{1+\beta\cos\pi r^2}{1+\beta}\ J_0(zr)\ r\ dr\ \right]^2 \quad \ldots \quad (6)$$

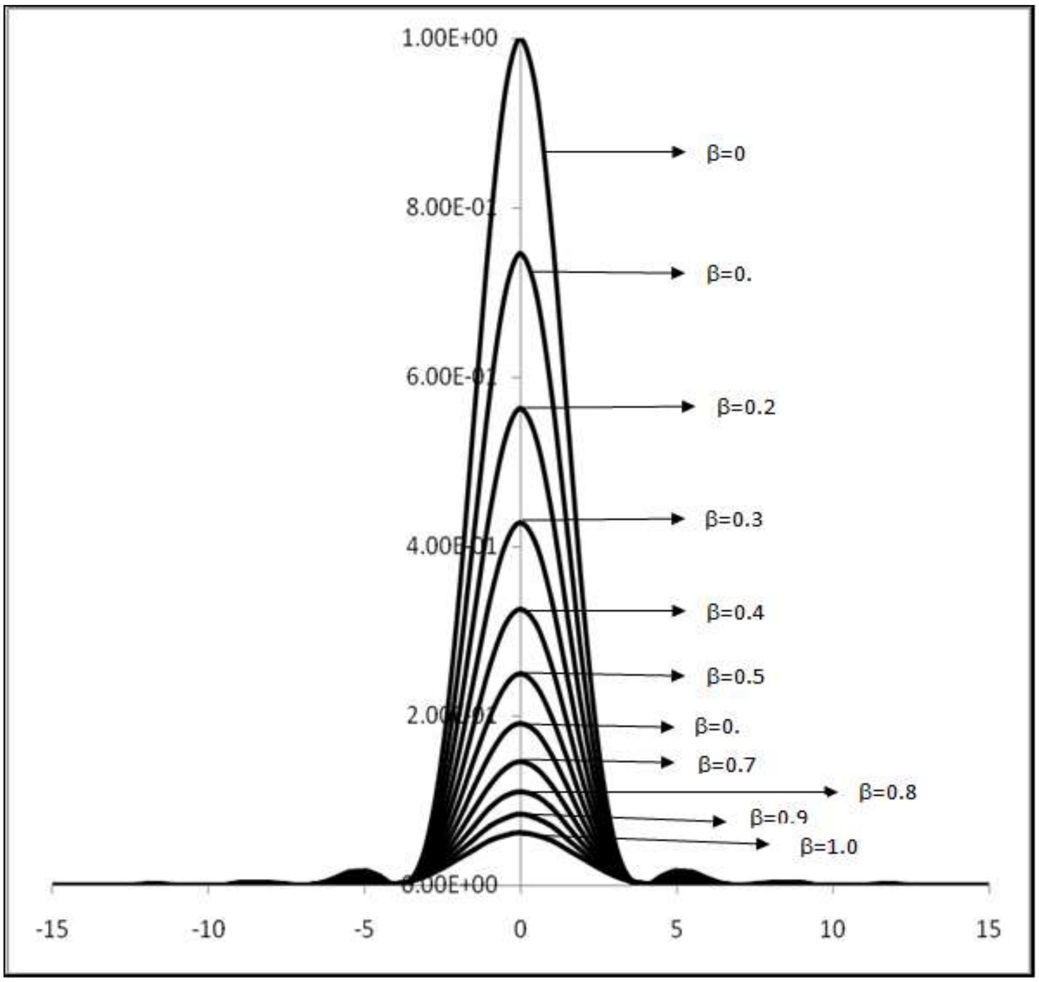

Fig. 2: Intensity PSF curves for various values of β.

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Table – 1
values of IPSF at the focused plane of observation, y = 0

z	β=1.0	β=0.9	β=0.8	β=0.7	β=0.6	β=0.5	β=0.4	β=0.3	β=0.2	β=0.1	β=0.0
-10	0.00003	0.00003	0.00003	0.00003	0.00004	0.00004	0.00004	0.00005	0.00006	0.00006	0.00008
-9.5	0.00003	0.00004	0.00007	0.00010	0.00015	0.00022	0.00030	0.00042	0.00059	0.00082	0.00115
-9	0.00001	0.00004	0.00009	0.00016	0.00027	0.00042	0.00065	0.00096	0.00141	0.00205	0.00297
-8.5	0.00000	0.00002	0.00006	0.00014	0.00027	0.00048	0.00078	0.00122	0.00185	0.00278	0.00413
-8	0.00001	0.00000	0.00001	0.00006	0.00014	0.00030	0.00053	0.00090	0.00144	0.00224	0.00344
-7.5	0.00007	0.00003	0.00001	0.00000	0.00001	0.00004	0.00012	0.00024	0.00045	0.00079	0.00130
-7	0.00015	0.00014	0.00012	0.00011	0.00009	0.00007	0.00006	0.00004	0.00002	0.00001	0.00000
-6.5	0.00021	0.00026	0.00033	0.00041	0.00052	0.00065	0.00082	0.00104	0.00133	0.00171	0.00224
-6	0.00018	0.00031	0.00049	0.00075	0.00110	0.00158	0.00223	0.00312	0.00435	0.00607	0.00851
-5.5	0.00006	0.00019	0.00043	0.00080	0.00136	0.00217	0.00332	0.00496	0.00729	0.01061	0.01542
-5	0.00001	0.00001	0.00013	0.00040	0.00090	0.00170	0.00292	0.00474	0.00742	0.01135	0.01717
-4.5	0.00046	0.00022	0.00006	0.00000	0.00009	0.00040	0.00101	0.00206	0.00376	0.00642	0.01055
-4	0.00207	0.00171	0.00135	0.00100	0.00067	0.00037	0.00014	0.00001	0.00005	0.00035	0.00109
-3.5	0.00561	0.00564	0.00567	0.00570	0.00574	0.00579	0.00584	0.00590	0.00597	0.00606	0.00616
-3	0.01167	0.01305	0.01467	0.01660	0.01891	0.02172	0.02515	0.02944	0.03486	0.04185	0.05109
-2.5	0.02037	0.02438	0.02926	0.03524	0.04263	0.05186	0.06350	0.07841	0.09778	0.12343	0.15815
-2	0.03112	0.03900	0.04879	0.06104	0.07645	0.09601	0.12109	0.15364	0.19651	0.25396	0.33261
-1.5	0.04256	0.05504	0.07077	0.09071	0.11608	0.14861	0.19071	0.24580	0.31890	0.41753	0.55341
-1	0.05281	0.06971	0.09124	0.11873	0.15396	0.19943	0.25858	0.33636	0.44002	0.58044	0.77458
-0.5	0.05994	0.08006	0.10582	0.13886	0.18139	0.23644	0.30827	0.40298	0.52949	0.70123	0.93910
0	0.06250	0.08380	0.11111	0.14619	0.19141	0.25000	0.32653	0.42752	0.56250	0.74587	1.00000
0.5	0.05994	0.08006	0.10582	0.13886	0.18139	0.23644	0.30827	0.40298	0.52949	0.70123	0.93910
1	0.05281	0.06971	0.09124	0.11873	0.15396	0.19943	0.25858	0.33636	0.44002	0.58044	0.77458
1.5	0.04256	0.05504	0.07077	0.09071	0.11608	0.14861	0.19071	0.24580	0.31890	0.41753	0.55341
2	0.03112	0.03900	0.04879	0.06104	0.07645	0.09601	0.12109	0.15364	0.19651	0.25396	0.33261
2.5	0.02037	0.02438	0.02926	0.03524	0.04263	0.05186	0.06350	0.07841	0.09778	0.12343	0.15815
3	0.01167	0.01305	0.01467	0.01660	0.01891	0.02172	0.02515	0.02944	0.03486	0.04185	0.05109
3.5	0.00561	0.00564	0.00567	0.00570	0.00574	0.00579	0.00584	0.00590	0.00597	0.00606	0.00616
4	0.00207	0.00171	0.00135	0.00100	0.00067	0.00037	0.00014	0.00001	0.00005	0.00035	0.00109
4.5	0.00046	0.00022	0.00006	0.00000	0.00009	0.00040	0.00101	0.00206	0.00376	0.00642	0.01055
5	0.00001	0.00001	0.00013	0.00040	0.00090	0.00170	0.00292	0.00474	0.00742	0.01135	0.01717
5.5	0.00006	0.00019	0.00043	0.00080	0.00136	0.00217	0.00332	0.00496	0.00729	0.01061	0.01542
6	0.00018	0.00031	0.00049	0.00075	0.00110	0.00158	0.00223	0.00312	0.00435	0.00607	0.00851
6.5	0.00021	0.00026	0.00033	0.00041	0.00052	0.00065	0.00082	0.00104	0.00133	0.00171	0.00224
7	0.00015	0.00014	0.00012	0.00011	0.00009	0.00007	0.00006	0.00004	0.00002	0.00001	0.00000
7.5	0.00007	0.00003	0.00001	0.00000	0.00001	0.00004	0.00012	0.00024	0.00045	0.00079	0.00130
8	0.00001	0.00000	0.00001	0.00006	0.00014	0.00030	0.00053	0.00090	0.00144	0.00224	0.00344
8.5	0.00000	0.00002	0.00006	0.00014	0.00027	0.00048	0.00078	0.00122	0.00185	0.00278	0.00413
9	0.00001	0.00004	0.00009	0.00016	0.00027	0.00042	0.00065	0.00096	0.00141	0.00205	0.00297
9.5	0.00003	0.00004	0.00007	0.00010	0.00015	0.00022	0.00030	0.00042	0.00059	0.00082	0.00115
10	0.00003	0.00003	0.00003	0.00003	0.00004	0.00004	0.00004	0.00005	0.00006	0.00006	0.00008

Table – 2
Locations of various Maxima and Minima

β	Central Intensity	Location of First Minimum	Location of First secondary Maximum	Location of Second Secondary Maximum
0.0	1.0000	3.83 (0.00110)	5.00 (0.0172)	8.50 (0.0041)
0.1	0.7459	4.00 (0.00040)	5.00 (0.1140)	8.50 (0.0028)
0.2	0.5625	4.00 (0.00005)	5.00 (0.0074)	8.50 (0.0019)
0.3	0.4275	4.00 (0.00001)	5.50 (0.0050)	8.50 (0.0012)
0.4	0.3265	4.00 (0.00014)	5.50 (0.0033)	8.50 (0.0007)
0.5	0.2500	4.00 (0.00040)	5.50 (0.0022)	8.50 (0.0005)
0.6	0.1914	4.50 (0.00009)	5.50 (0.0014)	9.00 (0.0002)
0.7	0.1462	4.50 (0.00000)	5.50 (0.0008)	9.00 (0.0001)
0.8	0.1111	4.50 (0.00006)	6.00 (0.0005)	9.00 (0.0001)
0.9	0.0839	5.00 (0.00001)	6.00 (0.0003)	9.50 (0.0000)
1.0	0.0625	5.00 (0.00001)	6.50 (0.0002)	9.50 (0.0000)

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III. RESULTS AND DISCUSSIONS

We have used the expression (6) above to evaluate the intensity PSF for the system under our consideration. As the expression (6) indicates, we shall discuss the results for the Intensity PSF in the focused plane of observation, i.e., y = 0, only. In the table-I, we have presented the computed values of the Intensity Point Spread Function (IPSF) for various values of the apodisation parameterβ. The results for diffraction-limited Airy pupil corresponding to a clear Circular aperture. In the Fig.2, we have graphically presented the variations in the IPSF for various values of β. It is observed from the curves that as the value of β is increased, there is a considerable drop in the value of the normalized central intensity. As a result, there is a broadening of the central diffraction maximum with the relative intensity values for a particular value of β.

In the table-2, we have shown the locations of the various Maxima and Minima in the intensity PSF. This information is very important while studying the Two-point Resolution capabilities of the optical system, according to the Rayleigh criterion. These positions are indicated clearly in the table and, therefore, do not need any further discussions.

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