Applications of Graph Theory in Network Analysis

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Abstract

A graph G=(V,E) is an ordered pair where V is the finite non empty set of vertices and E is the set of edges. A graph is represented by vertices as dots and edges as line segments. Here we try to analyze and provide a solution to the network using graph theoretical properties.

Keywords: Eulerian cycle and path, Representation of electrical networks using Graphs, Graph models representation, Matrix representation of graphs

I. INTRODUCTION

A connected acyclic graph called tree was implemented by G. Kirchhoff in 1847 and he used graph theoretical concept in the calculation of currents and voltages in networks and was later improved by J.C. Maxwell in 1892. An electrical network is a combination of active components, passive components or both, voltage source, current source, switches and loads interconnected electrically. The network components are idealized of physical device and system, in order to for them to represent several properties, they must obey the Kirchhoff’s law of currents and voltages. A graph representation of electrical network in terms of line segments or arc called edges or branches and points called vertices or terminals.

II. DEFINITIONS

A. Network:
It is an interconnection of electrical components like resistors, capacitors, inductors, diodes, transistors, voltage sources, current sources, switches and loads but not closed.

B. Circuit:
A closed network is called a Circuit.

C. Walk:
A Walk is an alternating sequence of vertices and edges which starts with a vertex and ends with a vertex and each edge ei joins the vertices v_i-1 and v_i. A walk with n vertices is denoted by v_o – v_n walk.

D. Path:
A walk in which no vertex is repeated is called a path.

E. Cycle:
A closed path is called a cycle.

F. Eulerian Graph:
A graph is said to be Eulerian if it covers all the edges of the graph.

G. KVL:
It is the abbreviated form of Kirchoff’s Voltage Law. It states that the algebraic sum voltages in a circuit is always zero, which means that the sum of voltage rise in a circuit is always equal to the sum of voltage drop in the circuit.
II. KCL:
It is the abbreviated form of Kirchoff’s Current Law. It states that the algebraic sum currents in a junction in a circuit is always zero, which means that the sum of incoming currents in a junction in a circuit is always equal to the sum of outgoing currents in the junction in the circuit.

III. REPRESENTATION OF ELECTRICAL NETWORKS USING GRAPHS
An electrical network can be represented using graphs as each node of the network can be considered as a vertex of the graph and each branch of the network can be considered as an edge.

It is easy to solve a network to find out the loop currents and node voltages in a circuit when the circuit is simpler. But when the degree of difficulty increases in the circuit, we should move to an alternative by compulsion. So we try to represent an electrical network with the help of a graph and thereby we try to solve the problem with reduced difficulties.

A. Network:

IV. A GRAPH MODEL REPRESENTATION
A graph model is used to represent a network by using the vertices and edges that cover the entire network. Given below is the graph model of the network shown above.
V. MATRIX REPRESENTATION OF GRAPHS

One of the important usages of graph theory is matrix representation of graphs. There are two main representations namely adjacency matrix and incidence matrix of a graph.

Let \( v_i \) and \( v_j \) be any two vertices of a graph \( G \). Adjacency matrix of the graph \( G \) denoted by \( A(G) \) is defined as the square matrix whose entries are either 0 or 1 defined by

\[
a_{ij} = \begin{cases} 
1 & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\
0 & \text{if } v_i \text{ and } v_j \text{ are not adjacent}
\end{cases}
\]

Let \( G \) be a graph with \( m \) vertices and \( n \) edges. Incidence matrix denoted by \( I(G) \) is defined as an \( n \times m \) matrix whose entries are given by

\[
a_{ij} = \begin{cases} 
1 & \text{if the } e_i \text{ is incident with } v_j \\
0 & \text{if the } e_i \text{ is not incident with } v_j
\end{cases}
\]

Now, considering the network the adjacency matrix can be constructed with the vertex set \( V = \{1, 2, 3, 4, 5, 6\} \) and edge set \( E = \{R1, R2, R3, R4, R5, R6, C1, C2, V1, S1, L1, L2\} \).

We shall construct a table from which we can construct the adjacency matrix easily.

<table>
<thead>
<tr>
<th>( v_i )</th>
<th>( v_j )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
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<tr>
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<td>0</td>
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<td>1</td>
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<tr>
<td>3</td>
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<td>0</td>
<td>0</td>
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<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>1</td>
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<td>0</td>
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<tr>
<td>5</td>
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<td>6</td>
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<td>7</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Also by constructing a table we can easily arrive at the incidence matrix.

<table>
<thead>
<tr>
<th>( e_i )</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<tbody>
<tr>
<td>R1</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>R2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
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</tr>
<tr>
<td>R3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>R4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>R5</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>R6</td>
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<td>1</td>
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<td>0</td>
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<tr>
<td>R7</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>0</td>
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<td>1</td>
<td>0</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>L1</td>
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<td>0</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>L2</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>V1</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>S1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

The incidence matrix of the graph is given by
From the adjacency and incidence matrix, the assumed loop currents and the node voltages of the electrical network can be calculated.

**VI. Conclusion**

The graph of a network plays a fundamental role in the study of circuits. Till now we have been focusing on providing graph theoretical approach to an electrical network. Graph theory has greater application in wide range of fields.

**References**

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